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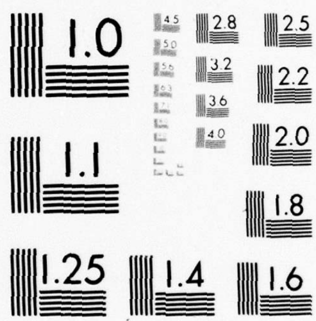


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# THESIS

FATIGUE CRACK PROPAGATION ANALYSIS
OF AIRCRAFT STRUCTURES

by

Larry Don Newsome

March 1978

Thesis Advisor: G. H. Lindsey

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Complete FORTRAN computer program input documentation for the IBM 360/67 system has been included as an appendix to enable this thesis to serve as a user's manual for CRACK'S II, an Air Force crack propagation program for aircraft fatigue damage.

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# Fatigue Crack Propagation Analysis of Aircraft Structures

by


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Lieutenant, United States Navy
B.S., Morehead State University, 1969


Submitted in partial fulfillment of the requirements for the degree of

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March 1978

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George J Haltiner, Dean of Science and Engineering

## ABSTRACT

This thesis is a comparative study of aircraft fatigue life calculations based upon crack propagation and upon cumulative damage. The stress concentration factor, which supplies sufficient geometric information for Miner's Law of cumulative damage, is found to not completely specify the geometry for the crack propagation approach. Effects on fatigue life of variations in initial crack length, plate width, hole size, and hole geometry for the same stress concentration factor have been investigated; also both ordered and random load histories were used to compare the two approaches.

Complete FORTRAN computer program input documentation for the IBM 360/67 system has been included as an appendix to enable this thesis to serve as a user's manual for CRACK'S II, an Air Force crack propagation program for aircraft fatigue damage.

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## LIST OF SYMBOLS

| | |
|---|---|
| $a$ | half-crack length |
| $a_c$ | crack length anytime following overload |
| $a_n$ | elliptical parameter |
| $a_o$ | initial half-crack length |
| $a_p$ | total affected crack length |
| $a_r$ | crack length after r loads |
| $a_p - a$ | distance from crack tip to elastic-plastic interface |
| $b$ | plate half-width |
| $b_n$ | elliptical parameter |
| $c$ | material constant |
| $C_p$ | retardation parameter |
| $da/dn$ | crack propagation rate |
| $K$ | stress intensity factor (psi $\sqrt{in}$) |
| $K_{ap}$ | applied stress intensity factor |
| $K_c$ | material fracture toughness (psi $\sqrt{in}$) |
| $K_{max}$ | maximum stress intensity factor (psi $\sqrt{in}$) |
| $K_{min}$ | minimum stress intensity factor (psi $\sqrt{in}$) |
| $K_T$ | stress concentration factor |
| $\Delta K$ | stress intensity range ($K_{max} - K_{min}$) |
| $K_Q$ | material constant |
| $K_I$ | applied crack tip stress intensity factor |

| | |
|---|---|
| $l_c$ | characteristic length |
| m | shaping function |
| n | material constant |
| r | number of loads applied or radius of circular hole |
| R | stress ratio ( $\sigma_{min}/\sigma_{max}$) |
| $R_{cutoff}$ | maximum value of R |
| $R_y$ | extent of current yield zone |
| t | material thickness |
| $\beta$ | correction factor |
| $\delta$ | $b_n/a_n$ |
| $\sigma_{max}$ | maximum applied tensile stress |
| $\sigma_{min}$ | minimum applied tensile stress |
| $\sigma_y$ | applied stress |
| $\Delta\sigma$ | stress range ( $\sigma_{max} - \sigma_{min}$) |

## I. INTRODUCTION

Pre-existing flaws from fabrication operations, or flaws generated in service, (cyclic loadings, nicks, dings, punctures produced by projectiles, etc.) have a significant effect on the life of an aircraft. The service life of an aircraft can be analyzed by calculating the total crack growth which can be tolerated prior to the formation of a critical size crack (maximum crack length at fracture under operational load). The Air Force has developed an analytical tool which can predict this growth under variable amplitude loading, leading to the critical crack length, associated safe operating periods, and inspection intervals.

A computer program (Cracks II) has been developed (Ref. 1) to facilitate calculation of the crack growth rate using various models (Forman, Paris, etc.). The crack growth rate is affected by the applied stresses, as well as by the residual stresses remaining after the application of a load. The residual compressive stresses lower the magnitude of the next applied stress. For this reason, Cracks II uses a retardation model (either Willenborg or Wheeler models) to more accurately predict aircraft service life.

The objectives of this thesis were to convert Cracks II from the IBM 7044/7094 Direct Coupled System (DCS) to the IBM 360/67 system, compare Cracks II service life prediction methods and times to

Miners' Cumulative Damage (Ref. 2) methods, and to create a workable method of predicting aircraft service life by analyzing cyclic loadings.

Several types of spectra, including random spectra, were used in the life prediction method. It should be noted that during all calculations the models were theoretical, not directly measurable or observable; however, the models have been proven to be effective in service life prediction in other analyses.

To confirm the validity and accuracy of Cracks II on the IBM 360/67 computer, a sample problem (Ref. 1) was prepared and compared exactly with a computer run sent from the Air Force Flight Dynamics Laboratory, Wright Patterson Air Force Base, Dayton, Ohio.

## II. DESCRIPTION OF CRACKS II

### A. CRACK PROPAGATION RATE MODELS

In the early 1960's, P.C. Paris (Ref. 3) determined that the rate of crack propagation under cyclic loading is directly related to the stress-intensity-factor range, $\Delta K$ ( $\Delta K = K_{max} - K_{min}$). Paris developed an exponential relationship by fitting experimental data in the following form:

$$\frac{da}{dN} = C\ (\Delta K)^n \qquad (1)$$

In 1967, Foreman, Kearney, and Engle published a paper (Ref. 4) in which Paris' equation was modified to take into account the effects of load ratio, R, and crack growth instability as $K_{max}$ in $\Delta K$ approached $K_c$. These modifications led to the following relationship:

$$\frac{da}{dN} = \frac{C\ (\Delta K)^n}{(1-R)K_c - \Delta K} \qquad (2)$$

Equation (2) is more commonly known as Forman's equation.

Other crack growth prediction models are options in Cracks II. One known as Walker's Equation is available, but it is not as well known as Forman's Equation and was not used in this thesis evaluation. It has the form:

$$\frac{da}{dN} = C\,[(1-R)^m K_{max}]^n \qquad (3)$$

Another model provides for a method of directly inputting tabular data of the form:

$$\left.\frac{da}{dN}\right]_i \quad Vs\ (\Delta K)_i$$

This method could be used if experimental data were available.

Cracks II provides the capability of modeling load interaction effects of crack growth retardation due to plasticity. Two models are available, the Wheeler model (Ref. 5) and the Willenborg model (Ref. 6). The Wheeler retardation model is used to modify any of the crack growth models (Forman, Paris, etc.). Wheeler's model takes the form:

$$a_r = a_o + \sum_{i=1}^{r} C_{pi} f(\Delta K_i) \tag{4}$$

where $C_{Pi}$ is the retardation parameter.

$a_r$= total crack length after r load applications.

$f(\Delta K_i)$= crack growth prediction method (eq(1), (2), or (3)),

and (see Fig. 1),

$$C_p = \left(\frac{R_y}{a_p - a}\right)^m \quad ; \quad a + R_y < a_p \tag{5}$$

where $R_y$ = extent of current yield zone.

$a_p - a$ = distance from crack tip to elastic-plastic interface.

m = shaping exponent.

The Willenborg retardation model does not operate directly on $\frac{da}{dN}$, as the Wheeler model does, instead it operates on $\Delta K$ in equations (1), (2), and (3) (see Fig. 2). It takes the form:

$$K_{ap} = \sigma_{ap} \sqrt{\pi a_i} \tag{6}$$

where $$\sigma_{ap} = \sigma_y \sqrt{\frac{C(a_p - aa}{a_c}} \tag{7}$$

# Wheeler Model

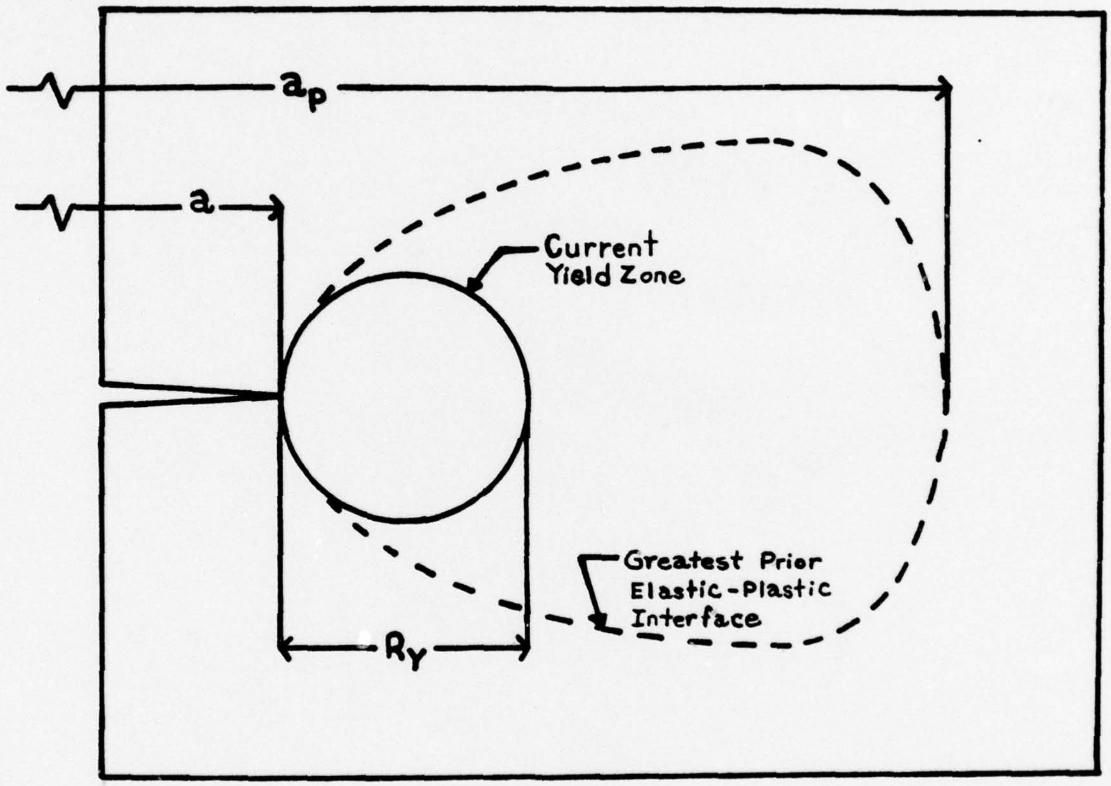


FIGURE 1

## Willenborg Model

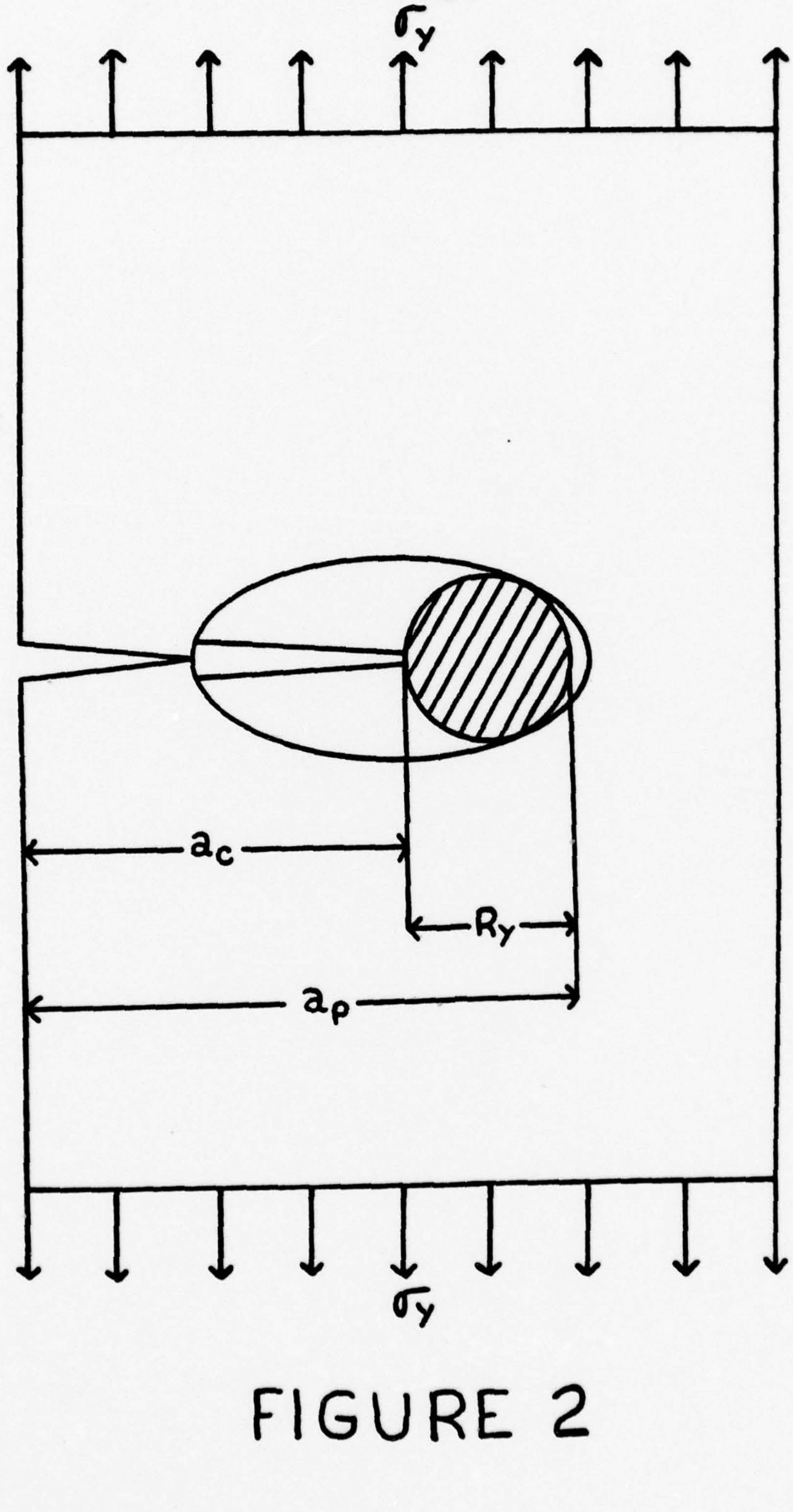


FIGURE 2

$a_c$ = crack length at anytime following overload

$\sigma_y$ = applied stress

$a_p$ = total affected crack length

C = material constant

$K_{ap}$ = applied stress intensity factor

In the analysis presented here, Forman's method, equation (2), and the Willenborg retardation model, equation (6), were used.

B. STRESS INTENSITY FACTOR

The basic parameter in fracture mechanics is the stress intensity factor, K. For opening mode crack propagation analysis, the applied crack tip stress intensity factor, $K_1$, must be less than the materials' fracture toughness, $K_c$, or fracture will occur. This applied crack tip stress intensity factor, $K_1$, is a function of geometry and loading type. For a central crack in an infinite sheet, the stress intensity factor may be written as follows:

$$K_1 = \sigma \sqrt{\pi a} \tag{8}$$

Equation (8) takes different forms depending upon the geometry and loading. In Cracks II these effects are treated as modifiers, or corrections, to equation (8). Thus a more general form of equation (8) is:

$$K_1 = \sigma \sqrt{\pi a}\ \beta \tag{9}$$

where $\beta$ is the correction factor (see part c). In the literature some authors delete $\pi$ from equations (8) and (9) altogether. For this reason, both options are available in Cracks II.

The stress intensity factor range, $\Delta K$, is defined as:

$$\Delta K = K_{max} - K_{min} \tag{10}$$

thus equation (9) becomes:

$$\Delta K = \Delta\sigma \sqrt{\pi a}\,\beta \tag{11}$$

where $\Delta\sigma = \sigma_{max} - \sigma_{min}$.

C. CORRECTION FACTORS

Equation (11) represents stress-intensity-factor ranges for a centrally located crack in an infinite panel, where $\beta$ is unity. For other geometries $\beta$ takes on different values. The $\beta_i$ available in Cracks II are as follows:

BETA 1.0 = constant multiplier.

This provides the analyst with the capability to scale loads or modify $\Delta K$ by a constant factor.

BETA 2.0 = Finite width tangent function.

This corrects for a finite width plate. The form of this correction is:

$$\beta_2 = \sqrt{\frac{2b}{\pi a} \tan\left(\frac{\pi a}{2b}\right)} \tag{12}$$

where b and a are shown in figure 3.

BETA 3.0 = tabular correction factor.

This permits the analyst to apply correction factors, which appear in the literature as in figure 4, as discrete data. The form of this correction is:

$$\beta_3 = f\ ({}^{a}/{l_c}) \tag{13}$$

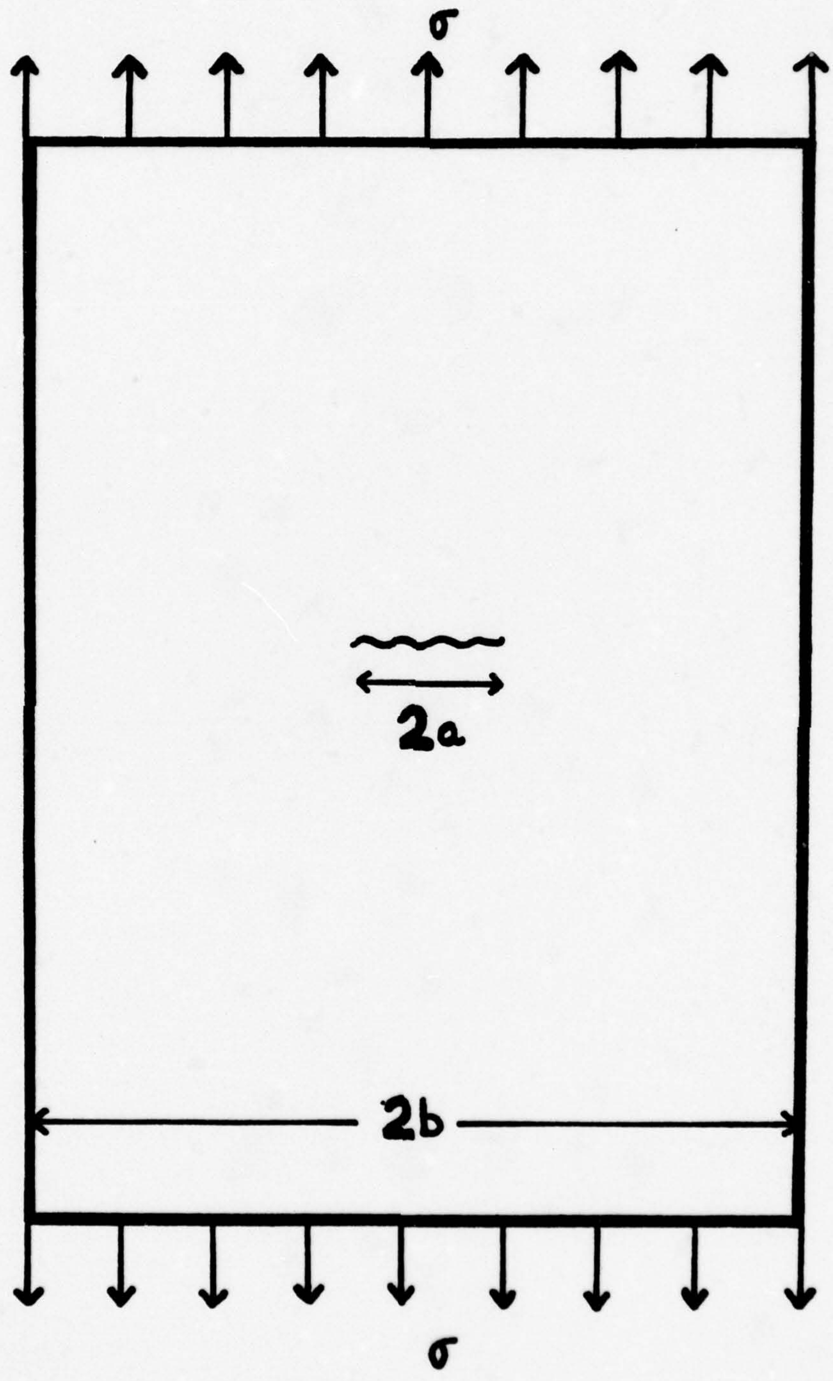


FIGURE 3

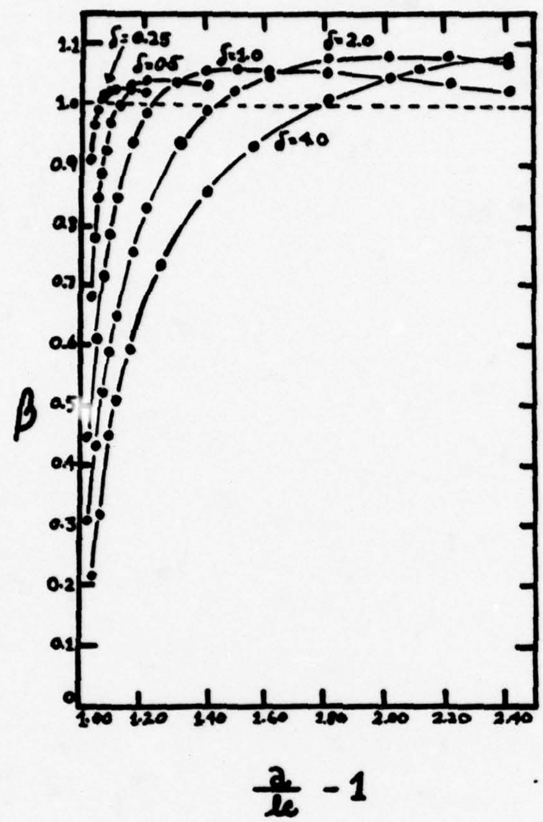


FIGURE 4

$$K = \beta \sigma \sqrt{\pi a}$$

where $\frac{b_n}{a_n} = \delta$

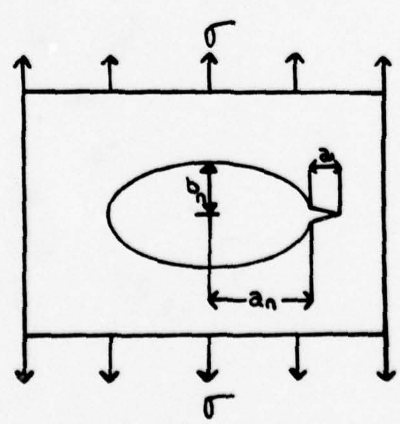


FIGURE 5

An example would be a crack emanating from an elliptical hole (see figure 5).

BETA 4.0 = alternate tabular correction factor.

This permits the analyst to restart the program with a different tabular correction factor, if necessary.

BETA 5.0 = Bowie solution for one crack from a circular hole.

This is one of the most common correction factors used in aircraft analysis due to numerous rivet holes, access holes, etc. The correction factor takes the form:

$$\beta_5 = 0.6762062 + \frac{0.8733015}{0.3245442 + a/b} \qquad (14)$$

where a and b are shown in figure 6.

BETA 6.0 = Bowie solution for a double crack from a circular hole.

This correction factor takes the form:

$$\beta_6 = 0.9438510 + \frac{0.6805078}{0.2771965 + a/b} \qquad (15)$$

where a and b are shown in figure 7.

It should be noted that for BETA 5.0 or BETA 6.0 that b does not have to be a finite number. If b is infinitely large then the factor $a/b \longrightarrow 0$.

For various combinations of geometries, BETA will become combinations of BETA (I). For example, if an analysis is to be made on a crack emanating from a circular hole (BETA 5.0), see figure 6, in a finite-width plate (BETA 2.0), BETA would be the product of BETA 2.0 and BETA 5.0. Therefore, in general,

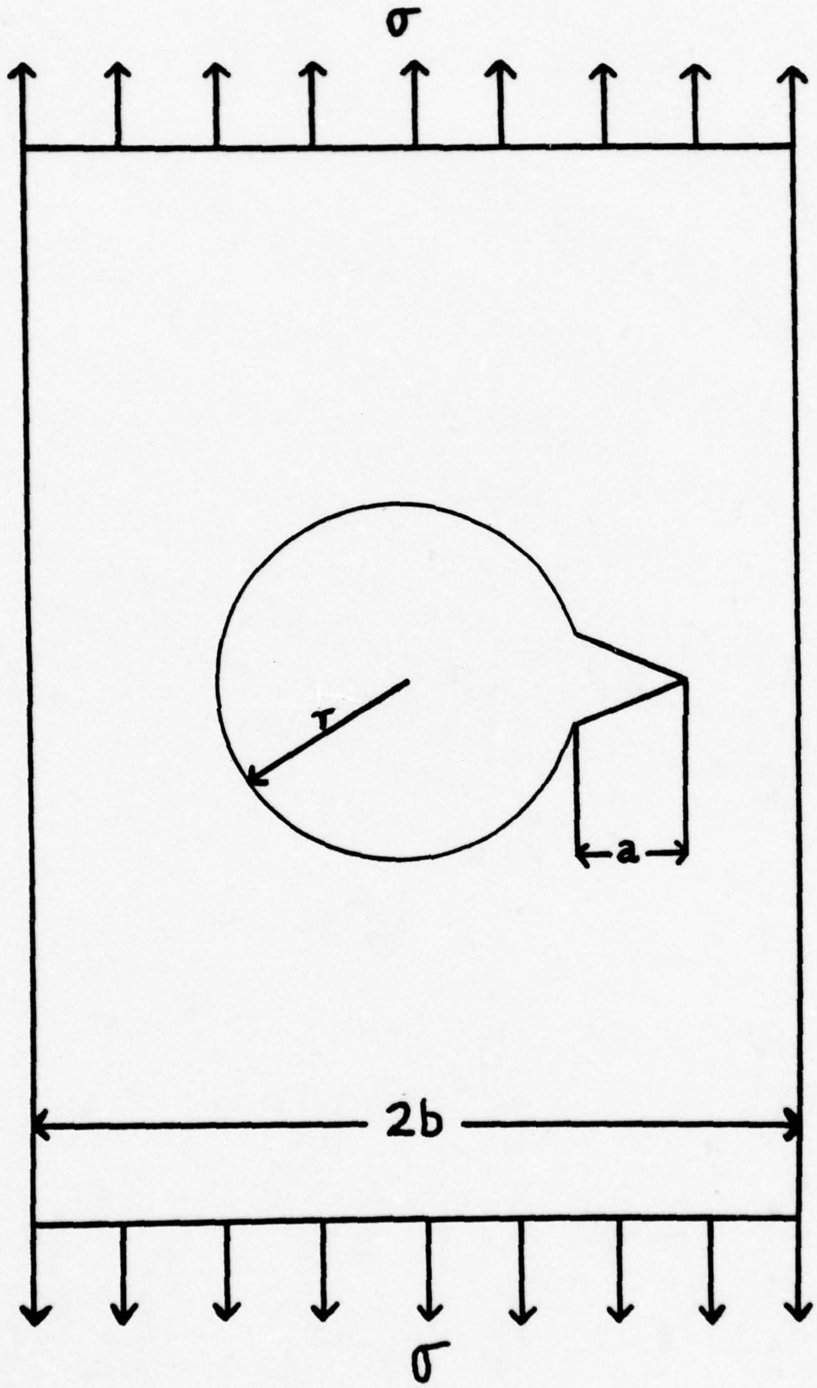


FIGURE 6

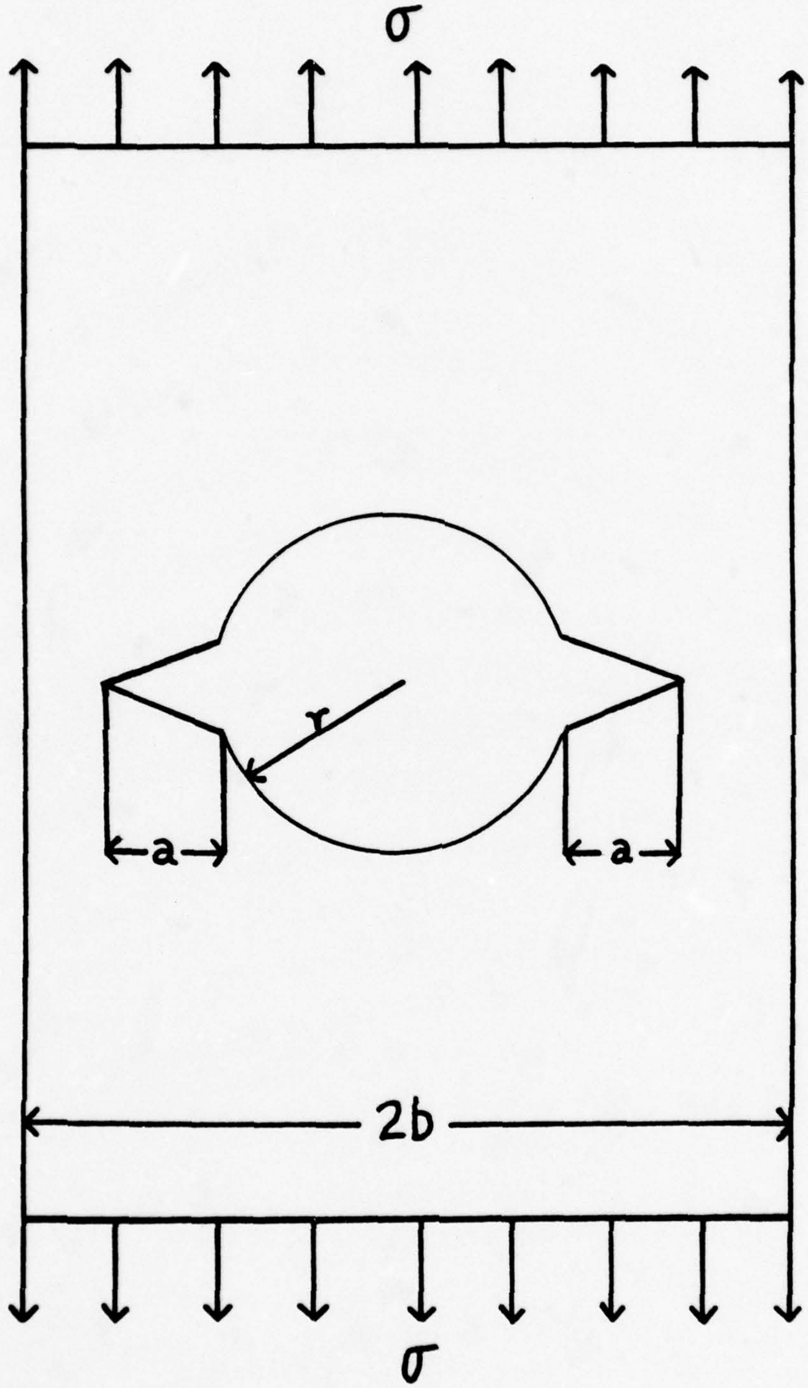


FIGURE 7

$$\beta = \prod \beta_i \tag{16}$$

This thesis is concerned with three of the corrections factors; BETA 2.0, BETA 3.0, BETA 5.0 and combinations thereof.

## D. INPUT DATA REQUIREMENTS

### 1. Label cards

The input requirements mentioned here are very general but are specific for a few cases. For more detailed input/output instructions to Cracks II, see reference 1 and the appendix.

The input consists of sections of data, which are identified by preceding label cards. These label cards are shown in Table I and must be typed exactly as shown. In certain label cards there are variables which take on different values depending on the option chosen. These values are defined in reference 1 and the appendix.

The purpose of the labeled input is to facilitate the parametric restart capability (discussed later). Each section is accessible individually and may be changed without affecting any other parameters. This is true even in the ANALYSIS section where changes in any of the BETA cards do not affect the ones which remain unchanged. However, this is not true in the LOADS section. Any change in the LOADS section results in complete redefinition of the mission segments. While this does not necessarily redefine the sequence of application (SPECTRUM), it may cause redefinition of the mission sequence; i.e., the order of the cards within the SPECTRUM section.

TABLE I

| | | |
|---|---|---|
| TITLE | - | the card immediately following this card is a number giving the number of cards in the title. The next card(s) are the title of the output/input. |
| EQUATION | - | the card following this label card tells which crack propagation method (Forman, Paris, etc.) is used. |
| MATERIAL | - | the card immediately following this label card usually tells where the material constants come from but can say anything you choose. The next card(s) list the material constants of the particular material being analyzed. |
| THRESHLD | - | the card following this card gives a threshold value for the stress intensity factor. |
| LIMITS | - | the card following this card gives the initial crack length, final crack length (usually omitted due to large default option), initial cycle, and R cutoff (R cutoff = 0.8 in this thesis). |
| ANALYSIS | - | this label card alerts the program that certain modifying cards are next. The ANALYSIS section must end with an END card. |
| | Note: | The following three label cards are part of the ANALYSIS section. |

TABLE I (cont'd)

| | |
|---|---|
| SURFACE - | This card indicates a surface flaw modifier, also on this card are the material thickness, t, and the initial surface half-crack length, $C_o$. |
| RETARD - | This card tells which type of retardation model, if any, will be used (Wheeler or Willenborg). Also, the card will indicate either plane stress or plane strain. |
| BETA - | This card indicates the geometry of the structure and associated correction factor (i.e., finite width, circular hole, etc.) |
| LOADS - | This card alerts the program that the stresses are to be inputted in one of three forms indicated by a Lodlab card. The card immediately following the LOADS card tells how many blocks of data are to be run and also the Spectrum title (anything you might choose). The LOADS section must end with an END LOAD card. |
| Note: | The following three cards are part of the loads section and are known as Lodlab cards and each part of the following three sections must contain an END card. |
| MAX-MIN - | This alerts the program that the following cards contain load input in the form of maximum stresses and minimum stresses. Each card contains one maximum stress and its associated minimum stress. |

TABLE I (cont'd)

| | |
|---|---|
| MAX-MIN (cont'd) - | After the stresses have been inputted, an END card signifies input in this form has ceased. A mission title can also be written on the MAX-MIN card itself, if necessary. |
| R-DELTA - | This card alerts the program that the following cards contain load input in the form of the difference between the maximum stress and the minimum stress ($\Delta\sigma = \sigma_{max} - \sigma_{min}$) and the stress ratio, R, ($R = \sigma_{min} / \sigma_{max}$). A mission title can also be written on the R-DELTA card itself. This part must end with an END card. |
| MEAN-ALT - | This card alerts the program that the following cards contain load input in the form of the mean stress and the alternating stress. A mission title may be written on the MEAN-ALT card itself. This part must end with an END card. |
| END LOAD - | This card indicates the end of the LOADS section. |
| SPECTRUM - | This card indicates that the flight profiles are to follow. The card immediately following the SPECTRUM card tells how many flights will be run. The next cards will be the individual flight profiles. For example, if there were 7 flights, one might want one pass on each flight except flight number 3. If a 4 is indicated |

TABLE I (cont'd)

| | |
|---|---|
| SPECTRUM - (cont'd) | on flight 3, then 4 passes will be made on this flight prior to going on to flight 4, etc. |
| RESTART - | This card allows you to rerun the same program with the only changes being the ones following the RESTART card. This is true for every change except the BETA card. In case of the BETA card, they would be treated as in equation (16). |
| PRINT - | This card indicates the level of output generated; for instance, crack length printed after each block or $\Delta K$, $K_{max}$, accumulated cycles, crack length printed for each layer in the program, etc. |
| END DATA - | This card signifies the end of the input. If the RESTART card is used, the END DATA card will be used again. |

Certain sections of the input have default options within the program. These default options provide values for certain key parameters, which enable the program to execute in certain instances when sections of input are inadvertently omitted. There is no default for the LOADS or SPECTRUM sections. These must always be present in the basic data package. The default options correspond to a Forman equation (eq. 2) formulation for 7075-T6 aluminum with no retardation. The ANALYSIS section, if omitted, produces a stress intensity formulation for a central crack in an infinite sheet. The default on the PRINT section gives crack growth at the end of each block in the spectrum.

2. Random Data Input

In addition to the Lodlab card mentioned in Table I, a method of generating random oriented stresses was used in Cracks II. This method is a modification to the original program.

When randomized stresses are inputted into Cracks II, another subroutine, RANDU, is used. This subroutine is a random number generator and is part of the subroutine library at the Naval Postgraduate School. An additional program was written (Ref. 2) to convert the random numbers into randomized stress input. This input is made directly into the subroutine, INPUT, in Cracks II. The MAX-MIN Lodlab card was used in conjunction with the input in order to have the stresses in the form of maximum stresses and minimum stresses. This, however, is not binding. Either of the Lodlab cards could have

been used. If any of the other Lodlab cards are used, the program must be rewritten in the subroutine, INPUT.

A comment card was inserted in the subroutine, INPUT, to indicate exactly where the RANDU section is to be placed and what other cards are to be removed. The change in the LOADS section of the input data is simply that there are no cards containing stresses following the Lodlab card, MAX-MIN. The card following the MAX-MIN card will be an END card rather than the maximum and minimum stresses.

## III. CRACK GROWTH CALCULATION

### A. METHOD

The analysis conducted here uses Forman's equation, equation (2), and the Willenborg retardation model, equation (6). The analysis consisted of using wing station (WS) 32 on A-7 aircraft with a known stress concentration factor, $K_T$, equal to 2.72. The 100% limit load factor is 29,800 psi at WS 32.

Table II gives the load spectrum used and is a repeat from reference 2 and MIL-STD-8866. Table III reduces the 42,006 loads of Table II by a factor of 10 to 4,201 loads. This means that 4,201 cycles corresponds to 100 hours of flight time.

Also listed in Table III are the maximum and minimum stresses. The maximum stresses are generated by multiplying the limit load, LL, by the per cent of maximum limit load. In general:

$$\text{max stress} = \text{LL} \times (\text{per cent of max LL})$$

For example, the first entry is as follows:

$$.35 \times 29,800 = 10,430$$

The minimum stresses in Table III are generated by taking 11% of the maximum limit load. This corresponds to 1-g flight.

As the stresses listed in Table III are maximum and minimum stresses, they are used in conjunction with Lodlab card MAX-MIN in the LOADS section of Cracks II.

TABLE II

Number of times per thousand hours that load factor is experienced

| Per cent of maximum (positive) Limit Load factor | Number of cycles that load factor occurs |
|---|---|
| 35 | 17,000 |
| 45 | 9,500 |
| 55 | 6,500 |
| 65 | 4,500 |
| 75 | 2,500 |
| 85 | 1,360 |
| 95 | 440 |
| 105 | 150 |
| 115 | 40 |
| 125 | 16 |

TABLE III

Number of times per hundred hours that load factor is experienced

| Per cent of maximum Limit Load factor | Number of cycles per 100 hours | Maximum stress (psi) | Minimum stress (psi) |
|---|---|---|---|
| 35 | 1,700 | 10,430 | 3,278 |
| 45 | 950 | 13,410 | 3,278 |
| 55 | 650 | 16,390 | 3,278 |
| 65 | 450 | 19,370 | 3,278 |
| 75 | 250 | 22,350 | 3,278 |
| 85 | 136 | 25,330 | 3,278 |
| 95 | 44 | 28,310 | 3,278 |
| 105 | 15 | 31,290 | 3,278 |
| 115 | 4 | 34,270 | 3,278 |
| 125 | 2 | 37,250 | 3,278 |

The stress spectrum of Table III was run in 4 different variations. They are Hi-Lo, Lo-Hi, Hi-Lo-Hi and random. The results of randomizing the input agreed very closely with the Hi-Lo spectrum; therefore, the Hi-Lo spectrum was used in subsequent computer runs to minimize computer time.

Current USAF methods of service life prediction start with an initial crack length, $a_o$, of 0.05 inches. While this is quite a large initial crack to assume, it does have merit for factor of safety reasons.

The method used here is simply to use the stresses in Table III and run enough cycles to failure. The crack length growth takes the form:

$$a_r = a_o + \sum_{i=1}^{r} f(\Delta K_i) \tag{17}$$

where $a_r$ = crack length after r loads.

$a_o$ = initial crack length.

$f(\Delta K_i)$ = Forman's equation with the Willenborg retardation model.

r = number of loads applied (4,201 loads = 100 hours flight time).

There are 3 main ways a material fails. One is that $\Delta K$ exceeds $(1-R)K_c$ in equation (2), another is that the crack length exceeds the plate width, b, and the last is that the applied stress intensity factor exceeds the material fracture toughness value.

B. RESULTS

From reference 7 we have a way to relate the stress concentration factor, $K_T$, to the plate width and hole radius. That is:

$$K_T = \frac{3}{r/b+1} \tag{18}$$

where r and b are defined in figure 6.

For a stress concentration factor, $K_T = 2.72$, this corresponds to an r/b ratio of 0.1. This means that regardless of the actual plate width, b, and hole radius, r, the stress concentration factor, $K_T$, will always be equal to 2.72 just as long as the ratio, r/b remains 0.1. In cumulative damage theory as used in reference 2, when $K_T$ is specified, the fatigue life is determined, whereas with crack propagation theory this is not the case.

Table IV lists the number of cycles to failure (or flight hours to failure) for various plate widths and hole radii, keeping $K_T = 2.72$ (r/b =0.1). Table IV starts with an initial crack length $a_o$, of 0.05 inches (standard USAF methods).

Table V lists the number of cycles to failure (or flight hours) for various plate widths and hole radii, again keeping $K_T = 2.72$ (r/b = 0.1). This time the initial crack length, $a_o$, has been varied.

TABLE IV

For $K_T = 2.72$

| Plate half-width, b | hole radius, r | number of cycles to failure | (flight hours) |
|---|---|---|---|
| 5 | .5 | 4,201 | 100 |
| 4 | .4 | 4,201 | 100 |
| 3 | .3 | 12,603 | 300 |
| 2 | .2 | 16,804 | 400 |
| 1 | .1 | 33,608 | 800 |
| 0.5 | .05 | 51,216 | 1,100 |

TABLE V

For $K_T = 2.72$

| Plate half-width, b | hole radius, r | initial crack length, $a_o$ | number of cycles to failure | (flight hours) |
|---|---|---|---|---|
| 5 | .5 | .1 | 4,201 | 100 |
| 5 | .5 | .05 | 4,201 | 100 |
| 5 | .5 | .03 | 4,201 | 100 |
| 5 | .5 | .005 | 8,402 | 200 |
| 5 | .5 | .001 | 21,005 | 500 |
| 1 | .1 | .1 | 25,206 | 600 |
| 1 | .1 | .05 | 33,608 | 800 |
| 1 | .1 | .03 | 37,942 | 903 |
| 1 | .1 | .005 | 46,211 | 1,100 |
| 1 | .1 | .001 | 63,015 | 1,500 |
| 0.5 | .05 | .1 | 37,809 | 900 |
| 0.5 | .05 | .05 | 51,216 | 1,600 |
| 0.5 | .05 | .03 | 84,041 | 2,000 |
| 0.5 | .05 | .005 | 113,448 | 2,700 |
| 0.5 | .05 | .001 | 130,145 | 3,098 |

## IV. DISCUSSION OF RESULTS

### A. CIRCULAR HOLES

When analyzing Table IV it becomes apparent that the fatigue life (cycles to failure) is anything but fixed when the stress concentration factor, $K_T$, is specified. Also from Table IV, the fatigue life increases as the initial crack length, $a_o$, decreases. This seems intuitively correct, but recall that the stress concentration factor is still 2.72.

The cumulative damage theory used in reference 2 takes basically the same form as equation (17). However, the damage is associated with a specified stress concentration factor while the crack theory yields many different fatigue lives for the same stress concentration factor.

Cracks II uses a cumulative damage type of process in the form of a growing crack, but it does not rely upon the techniques of damage accumulation in the classic sense of Miner's Law as used in reference 2.

While the methods of reference 2 are viable with only a knowledge of stress concentration factor, Cracks II cannot be used in the same fashion i.e., by fixing the r/b ratio. A better way of using Cracks II is to model the actual hardware geometry and use the initial crack length, $a_o$, of 0.05 inches (for factor of safety reasons) and keep the plate widths and hole radii matched to existing values, (see Table VI).

TABLE VI

Circular holes

| Plate half-width, b | hole radius, r | initial crack length, $a_o$ | Number of cycles to failure (flight hours) | |
|---|---|---|---|---|
| 5 | .1 | .05 | 46,211 | 1,100 |
| 5 | .1 | .001 | 75,618 | 1,800 |

TABLE VII

Elliptical holes

| $a_n$ | $a_o$ | $\varsigma$ | Number of cycles to failure | Flight hours to failure |
|---|---|---|---|---|
| 0.43 | .05 | 1.16 | 105,025 | 2,500 |
| 0.43 | .001 | 1.16 | $\infty$ | $\infty$ |
| 2.0 | .05 | 1.16 | 365,487 | 8,800 |
| 2.0 | .001 | 1.16 | $\infty$ | $\infty$ |

## B. ELLIPTICAL HOLES

The stress concentration factor for elliptical holes takes the form:

$$K_T = 1 + \frac{2a_n}{b_n} \qquad (19)$$

where $a_n$ and $b_n$ are described in figure 5.

From figures 4 and 5 we have $\delta = b_n/a_n$. Again, a stress concentration factor of 2.72 is used, as in the circular holes. From this we get $\delta = 1.16$. This value of $\delta$ is used with figure 4 to obtain the value of the correction factor, $\beta$ . In this case BETA 3.0 is used for tabular input directly into Cracks II (see Table 3 of the appendix).

We see from Table VII that the life increases significantly as the value $a_n$ is increased, holding $\delta$ constant. This is intuitively correct, since the local stress concentration at the crack would be higher if the radius of curvature is smaller. The same reasoning applies when comparing circular holes with elliptical holes. The fatigue life is larger when dealing with elliptical holes for the same stress concentration factor, because the radius of curvature at the crack is larger for the cases studied.

## C. USE OF CRACKS II TO SET INSPECTION INTERVALS

Subsequent use of Cracks II for service life prediction could be as mentioned in part B of section III. For example, from Table VI, the number of cycles to failure is 46,211 (or 1,100 hours), for an initial crack length of 0.05 inches. This does not mean that the aircraft will fail

after only 1, 100 hours of flight time. It means merely that, on certain maintenance inspections (approximately every 1, 100 flight houts), WS 32 in the A-7 should be checked for a crack. If none appears, then it should be checked again on a subsequent maintenance check, after another 1100 hours.

## V. RECOMMENDATIONS

The analysis here deals with random and ordered data. To further substantiate the results, experimental data (flight test or laboratory) should be used in Cracks II and compared to known service lives. Data on the F-14 and F-18 aircraft have been promised for the near future from NAVAIRSYSCOM.

# VI. APPENDIX - INPUT DATA FORMAT

| Cols 1–10 | 11–20 | 21–30 | 31–40 | 41–50 | 51–60 | 61–70 | 71–80 |
|---|---|---|---|---|---|---|---|
| TITLE | | | | | | | |
| N | N=Number | of cards in title | | | | | |
| | | Any Information | | | | | |
| EQUATION | | | | | | | |
| EQN | Delk | (see table 1) | | | | | |
| MATERIAL | | | | | | | |
| | | Any Information | | | | | |
| Con1 | Con2 | Con3 | Con4 | Con5 | (see table 2) | | |
| $K_Q$ | $\sigma_y$ | | | | | | |
| THRESHLD | | | | | | | |
| $\Delta K_0$ | A | | $\Delta K_{th} = \Delta K_0 (1-A*R)$ | | | | |
| LIMITS | | | | | | | |
| $a_0$ | $a_f$ | $N_f$ | $R_{cut}$ | | | | |
| ANALYSIS | (see table 3) | | | | | | |
| SURFACE | $C_0$ | t | | | | | |
| RETARD | model | Yldzone | Con | No Retrd | Ovld | $a_0$ | |
| BETA | Con1 | Con2 | Con3 | Con4 | Con5 | | |
| END | | | | | | | |
| LOADS | (see table 4) | | | | | | |
| NbLKS (I10) | | Spectrum Title | | | | | |

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```
Lod lab  mission title
         Smax    Smin    Cycles
END
END LOAD
SPECTRUM
Nsegs        Nsegs-Number of segments in each block (1000 max)
Nflts Mission    Nflts-flights of mission(I) in this segment (Nsegs card)
RESTART
Istrt        (see table 5)
PRINT
IP1   IP2    IP3   IP4    Integer format (see table 6)
END DATA
```

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Table 1

| EQN | |
|---|---|
| FORMAN | $da/dN = C(\Delta K)^n / [(1-R)K_c - \Delta K]$ |
| WALKER | $da/dN = C[(1-R)^m K_{max}]^n$ |
| PARIS | $da/dN = C(\Delta K)^n$ Ignores R effects |
| DA/DN | $(da/dN)_i$ vs $(\Delta K)_i$ Ignores R effects |
| DELK | |
| | $K = \sigma\beta\sqrt{\pi a}$ |
| NASA | $K = \sigma\beta\sqrt{a}$ |

Table 2

| EQN | Con1 | Con2 | Con3 | Con4 | Con5 |
|---|---|---|---|---|---|
| FORMAN | C | n | $K_c$ | | |
| WALKER | C | m | n | | |
| PARIS | $C_1$ | $n_1$ | $\Delta K_{trans}$ | $C_2$ | $n_2$ |
| DA/DN | $N_{pts}$ | | | | |
| | $\Delta K_1$ | $da/dN_1$ | | | |
| | ⋮ | | | | |
| | $\Delta K_{npts}$ | $da/dN_{npts}$ | | | |

$K_Q$ = Cutoff K. If $K_{max} > K_Q$ terminate analysis

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| 1–6 | 7–10 | 11–20 | 21–30 | 31–40 | 41–50 | 51–60 | 61–80 |
|---|---|---|---|---|---|---|---|
| | | | | Table 3 | | | |
| | | Con1 | Con2 | Con3 | Con4 | Con5 | |
| BETA | | 1.0 | value | $a_i$ | $a_i$ | | Constant Factor |
| BETA | | 2.0 | b | $a_i$ | $a_i$ | | Finite Width |
| BETA | | 3.0 | pts | lca | $a_i$ | $a_i$ | Tabular |
| Tabular Title | | | | | | | |
| a/lc2 | | β | | | | | |
| ⋮ | | ⋮ | | | | | |
| BETA | 4.0 | | pts | lca | $a_i$ | $a_i$ | Alternate Table |
| same format as Beta 3.0 | | | | | | | |
| BETA | | 5.0 | r | $a_i$ | $a_i$ | | Bowie-fit 1 crack |
| BETA | | 6.0 | r | $a_i$ | $a_i$ | | Bowie-fit 2 cracks |
| | | | | | | | |
| | | | Variables | | | | |
| | | | | | | | |
| Co | | -surface half-crack length | | | | | |
| t | | -material thickness | | | | | |
| Yldzone | | -0-Plane stress; 1-Plane strain | | | | | |
| No retd | | -0-Repeat Solution ignoring retardation; 1-Supress unretarded solution | | | | | |

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Table 3 (cont.)

Model - 1.0 - Wheeler - con = m

- 2.0 - Willenborg - con = 0. - standard

con = olmax - generalized model

olmax - overload ratio for crack arrest

value - constant multiplier

b - Plate half-width

pts - Number of points in table

lc - Normalizing factor (usually some characteristic length)

r - Hole radius

ai, af - Apply correction factor from ai to af

Table 4

Lodlab Smax Smin

MAX-MIN ← $\sigma_{max}$ * $\sigma_{min}$ →

R-DELTA ← $\Delta\sigma$ * R → $\Delta\sigma = \sigma_{max} - \sigma_{min}$, $R = \sigma_{min}/\sigma_{max}$

MEAN-ALT ← $\sigma_{mean}$ * $\sigma_{alternating}$ →

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```
                                    Table 5

Istrt
    0      New Data to follow
    1      Modification to Current data case to follow

                                    Table 6

Print Options
       IP1  Print a, Δa, Δa/block - every Ith block
       IP2  Print a, Δa, Δa/segment - every Ith segment
       IP3  Print a, Δa, Δa/flight - every Ith flight
       IP4  Print a, ΔK, Kmax, da/dN - every Ith block (every layer in block)
```

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